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17MATDIP41

Fourth Semester B.E. Degree Examination, July/August 2021
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 3 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$$

by using elementary row operations. **(06 Marks)**

b. Solve the following system of equations by Gauss elimination method:
$x+y+z=9;\quad x-2y+3z=8;\quad 2x+y-z=3$ **(07 Marks)**

c. Find the inverse of the matrix $A=\begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$ using Cayley-Hamilton theorem. **(07 Marks)**

2 a. Show that eigen values of matrix $A=\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ are 0, 1, 1 and find eigen vector corresponding to the eigen value '0'. **(06 Marks)**

b. Test the following system for consistency and solve the system if the system is consistent
$x+2y+3z=1,\quad 2x+3y+8z=2,\quad x+y+z=3$. **(07 Marks)**

c. Using Cayley-Hamilton theorem, find the inverse of the matrix, $A=\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **(07 Marks)**

3 a. Solve $\dfrac{d^3y}{dx^3}-4\dfrac{d^2y}{dx^2}+5\dfrac{dy}{dx}-2y=0$. **(06 Marks)**

b. Solve $(D^2-13D+12)y=e^{2x}+5e^{x}$ **(07 Marks)**

c. Solve by using the method of undetermined coefficients: $\dfrac{d^2y}{dx^2}+y=2\cos x$. **(07 Marks)**

4 a. Solve $\dfrac{d^2x}{dt^2}-3\dfrac{dx}{dt}+2x=0$ given $x=0$ and $\dfrac{dx}{dt}=1$ when $t=0$. **(06 Marks)**

b. Solve $y''-4y'+4y=x^2+\cos 2x$ **(07 Marks)**

c. Solve by the method of variation of parameters $y''+y=\operatorname{cosec} x$ **(07 Marks)**

5 a. Find $L\{\sin t \cdot \sin 2t \cdot \sin 3t\}$. **(06 Marks)**

b. Find (i) $L\{e^{-3t}\cos 4t\}$ (ii) $L\left\{\dfrac{e^{at}-e^{bt}}{t}\right\}$ **(07 Marks)**

c. Find $L\{f(t)\}$ where $f(t)=\begin{cases}3t, & 0<t<2\\ 6, & 2<t<4\end{cases}$, given f(t) is the periodic function with the period 4. **(07 Marks)**

6 a. Find $L\{4+4^t+4\sin^2 t\}$ **(06 Marks)**

b. Find $L\{t^2 e^{3t}\sin t\}$ **(07 Marks)**

c. Express $f(t)=\begin{cases}\sin t, & 0<t<\pi\\ \cos t, & t>\pi\end{cases}$ interms of unit step function and hence find $L\{f(t)\}$. **(07 Marks)**

7 a. Find $L^{-1}\left\{\dfrac{1}{(s+1)(s+2)(s+3)}\right\}$. **(06 Marks)**

b. Find the inverse Laplace transform of $\log\left(\dfrac{s+a}{s+b}\right)$ **(07 Marks)**

c. Solve $y''+4y'+3y=0$ given $y(0)=0$, $y'(0)=1$ using Laplace transform. **(07 Marks)**

8 a. Find $L^{-1}\left\{\dfrac{s+1}{s^2+6s+9}\right\}$. **(06 Marks)**

b. Find inverse Laplace transform of $\cot^{-1}(s-a)$. **(07 Marks)**

c. Solve $y''+2y'+y=6te^{-1}$ under the conditions $y(0)=0$, $y'(0)=0$ by using Laplace transformation. **(07 Marks)**

9 a. Define conditional probability. Given for the events A and B, $P(A)=\dfrac{3}{4}$, $P(B)=\dfrac{1}{5}$ and $P(A\cap B)=\dfrac{1}{20}$, find $P\left(\dfrac{A}{B}\right)$, $P\left(\dfrac{B}{A}\right)$, $P\left(\dfrac{\overline{A}}{\overline{B}}\right)$, $P\left(\dfrac{\overline{B}}{\overline{A}}\right)$ **(06 Marks)**

b. Three students A, B, C, write an entrance examination. Their chances of passing are $\dfrac{1}{2}$, $\dfrac{1}{3}$ and $\dfrac{1}{4}$ respectively. Find the probability that
(i) atleast one of them passes
(ii) all of them passes
(iii) atleast two of them passes. **(07 Marks)**

c. Three machines A, B, C produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. If an item is selected at random, what is the probability that is defective? If a selected item is defective, what is the probability that is from machine A? **(07 Marks)**

10 a. State and prove Baye's theorem. **(06 Marks)**

b. A box contains three white balls and two red balls. If two balls are drawn in succession, find the probability that the first removed ball is white and the second is red. **(07 Marks)**

c. If a pair of dice is thrown what is the probability that
(i) the sum of numbers is divisible by 4
(ii) the number on the first is greater than that on the second. **(07 Marks)**

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